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## Dyonic $p$ -branes from self-dual $(p+1)$ -branes

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### ABSTRACT

The ‘electromagnetic’  $Sl(2; \mathbb{Z})$  duality group in spacetime dimension  $D = 4k$  can be given a Kaluza-Klein interpretation in  $D = 4k + 2$  as the modular group of a compactifying torus. We show how dyonic  $2(k - 1)$ -branes in  $D = 4k$  can be interpreted as self-dual  $(2k - 1)$ -branes in  $D = 4k + 2$  wound around the homology cycles of the torus. In particular, dyons of the D=4 N=4 heterotic string theory are interpreted as winding modes of a D=6 self-dual string, while D=8 dyonic membranes are interpreted as wound 3-branes of D=10 IIB superstring theory. We also discuss the T-dual IIA interpretations of D=8 dyonic membranes.

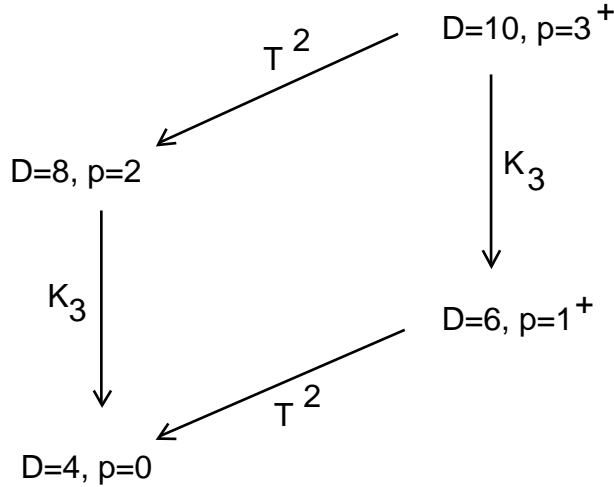
## 1. Introduction

It is now widely appreciated that extended objects, or p-branes, play an important part in our understanding of superstring dualities in various spacetime dimensions  $D$ . This paper is dedicated to a study of the role of dyonic p-branes in spacetime dimensions  $D = 4k$ , for integer  $k$ . The prototype is that of D=4 dyons of the  $\mathbb{T}^6$ -compactified heterotic string, and there is now strong evidence that they fill out multiplets of the S-duality group  $Sl(2; \mathbb{Z})$ . It was observed in [1] that this group can be interpreted as the modular group of the Kaluza-Klein (KK) torus arising from compactification on  $\mathbb{T}^2$  of a new self-dual D=6 superstring theory; a similar observation was made in [2] in the context of an abelian gauge theory. In the context of the effective D=6 field theory the self-dual string can be identified as a string soliton of the chiral N=4 supergravity obtained by  $K_3$ -compactification of D=10 IIB supergravity [3]; it is just the self-dual string soliton found in [4] as a solution of N=2 D=6 supergravity. One result of this paper is to show that the winding modes of this D=6 self-dual string around the homology cycles of the KK torus can be identified as D=4 dyons filling out S-duality multiplets.

In [1] the D=6 self-dual string theory was argued to result from the  $K_3$ -compactification of the D=10 type IIB superstring theory, the self-dual string itself being identified as a IIB 3-brane wrapped around a two-cycle of  $K_3$ . Thus, S-duality in string theory was shown to be a consequence of the non-perturbative equivalence of the  $\mathbb{T}^6$ -compactified heterotic string theory to the  $K_3 \times \mathbb{T}^2$  compactified IIB superstring theory. In this ‘10 to 6 and then to 4’ approach to heterotic S-duality, D=6 is merely a convenient intermediate dimension arrived at by consideration of the  $K_3$  compactification *prior* to a subsequent  $\mathbb{T}^2$  compactification. Clearly, one could perform the  $\mathbb{T}^2$  compactification first to arrive at an intermediate type II D=8 superstring theory. This ‘10 to 8 and then to 4’ approach was explored in [5], except that the D=10 starting point was there taken to be the IIA superstring theory. Here we concentrate on the IIB case, although we shall have more to say about IIA at the conclusion of the paper.

The D=8 type II superstring theory has an  $Sl(3; \mathbb{Z}) \times Sl(2; \mathbb{Z})$  U-duality group, with the  $Sl(2; \mathbb{Z})$  subgroup acting by ‘electromagnetic’ duality on the 4-form field strength (descending from the 4-form field strength of D=11 supergravity) whose sources are dyonic membranes. As we shall see, this  $Sl(2; \mathbb{Z})$  ‘electromagnetic’ duality group (which is also a subgroup of the  $Sl(2; \mathbb{Z}) \times Sl(2; \mathbb{Z})$  T-duality group) can be identified as the modular group of the torus arising from compactification to D=8 of the D=10 type IIB superstring, and the dyonic membranes as windings of the IIB 3-brane around the homology cycles of the torus. As explained in [5], upon further  $K_3$ -compactification the D=8 dyonic membranes can be wrapped around the homology 2-cycles of  $K_3$  to yield D=4 dyons which can be identified as those filling out S-duality multiplets in the equivalent heterotic string.

The two approaches to the interpretation of D=4 heterotic S-duality in terms of the D=10 IIB superstring theory are illustrated by the following diagram of commuting compactifications:



Whether one adopts the ‘10 to 6 and then to 4’ or the ‘10 to 8 and then to 4’ point of view, there is an  $Sl(2; \mathbb{Z})$  subgroup of the U-duality group in  $D = 4k$

dimensions, for either  $k = 1$  or  $k = 2$ , that can be interpreted as the modular group of the Kaluza-Klein torus for a  $\mathbb{T}^2$  compactification from  $D = 4k + 2$  dimensions. The main purpose of this paper is to show that a dyonic p-brane of the effective  $(4k)$ -dimensional field theory with (magnetic, electric) charge vector  $(m, n)$  can be interpreted as a  $(4k + 2)$ -dimensional self-dual  $(p+1)$ -brane wrapped  $m$  times around one homology cycle of the torus and  $n$  times around the other. Since the basic idea is applicable in spacetime dimensions  $D = 4k$  for arbitrary integer  $k$  we shall first develop it in this more general context.

Specifically, we consider an  $Sl(2; \mathbb{R})$ -invariant  $4k$ -dimensional field theory for a  $(2k)$ -form field strength  $F$ , and a complex scalar field  $\tau$  taking values in  $Sl(2; \mathbb{Z}) \backslash SL(2, \mathbb{R}) / U(1)$ . We shall show that this theory is the dimensional reduction on  $\mathbb{T}^2$  (and consistent truncation) of a self-dual  $(2k + 1)$ -form theory in a  $(4k + 2)$ -dimensional spacetime  $M_{4k+2} = M_{4k} \times \mathbb{T}^2$ , with the modular ‘parameter’ of  $\mathbb{T}^2$  identified with the complex scalar field of the  $D = 4k$  theory and the modular group of  $\mathbb{T}^2$  identified with the  $SL(2, \mathbb{Z})$  duality group of the  $D = 4k$  theory. We then apply this result for  $k = 1, 2$  to show, firstly, that dyons of D=4 N=4 supergravity can be lifted to D=6 as winding modes of a D=6 self-dual string and, secondly, that the dyonic membrane solutions of D=8 N=2 supergravity can be lifted to D=10 as winding modes of the self-dual IIB 3-brane. These results are reminiscent of the way in which the  $Sl(2; \mathbb{Z})$  multiplet of IIB strings [6] can be understood as winding modes of the D=11 membrane via a compactification to D=9 on  $\mathbb{T}^2$  [7], although the details are somewhat different.

It was shown in [5] that the D=8 dyonic membrane solutions can be lifted to D=11 to give new solutions of D=11 supergravity that interpolate between the membrane and 5-brane<sup>★</sup>. These solutions, which also preserve half the supersymmetry, might be viewed as the effective field theory realisation of membrane/5-brane bound states. Alternatively, the D=8 dyonic membrane solutions can be lifted to D=10 to yield solutions of IIA supergravity that might represent

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★ Thus, D=11 membrane/5-brane duality can be viewed as a manifestation of ‘electromagnetic’ duality in D=8; an elaboration of this point can be found in [8].

membrane/4-brane bound states. Remarkably, these membrane/4-brane solutions preserve half the supersymmetry and are *self-dual*. They can be found by reduction from D=11, which is the method used here to find them. We also use this method to find two other D=10 ‘bound state’ solutions, originally suggested in [9], which turn out to be duals of each other.

During the completion of this work a paper appeared [10] having some overlap with section 2 of this paper.

## 2. $\mathbb{T}^2$ -reduction of self-dual gauge theories

Let  $F = dA$  be a  $(2k)$ -form field strength and  $\tau$  a complex scalar field in a  $(4k)$ -dimensional spacetime,  $M_{4k}$ . Consider the action

$$I_{4k} = \int d^{4k}x \sqrt{-g} \left[ R - \frac{1}{2} \frac{d\tau d\bar{\tau}}{(\text{Im } \tau)^2} \right] + \alpha \int F \wedge G. \quad (2.1)$$

where

$$G = \text{Im } \tau {}^*F - \text{Re } \tau F \quad (2.2)$$

and  $\alpha$  is a real constant; for agreement with the conventions of [5] we must choose

$$\alpha = 2[(2k)!]^{-2}. \quad (2.3)$$

The asterisk in (2.2) indicates the Hodge dual with respect to the metric on  $M_{4k}$ .

The kinetic term for  $\tau$  is invariant under the  $SL(2; \mathbb{R})$  transformation

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}. \quad (2.4)$$

The  $A$ -field equations are also  $SL(2, \mathbb{R})$ -invariant provided that  $F$  transforms by

‘electromagnetic’ duality, i.e.

$$(F, G) \rightarrow (F, G)A^{-1} , \quad (2.5)$$

where  $A$  is the  $SL(2, \mathbb{R})$  matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} . \quad (2.6)$$

We now show how the action (2.1) can be obtained by compactification on  $\mathbb{T}^2$  of a  $(4k+2)$ -dimensional action. We do so by generalizing the method of [2] to apply to curved space backgrounds with Minkowski signature. We start from the  $D = 4k+2$  action

$$I_{(4k+2)} = \int d^{(4k+2)}x \sqrt{-g}R + \alpha \int [dC \wedge H + \frac{1}{2}H \wedge {}^*H] , \quad (2.7)$$

where  $C$  is a  $(2k)$ -form and  $H$  is a  $(2k+1)$ -form on a  $(4k+2)$ -dimensional spacetime  $M_{(4k+2)}$ . The matter field equations are

$$\begin{aligned} H - {}^*dC &= 0 \\ dH &= 0 , \end{aligned} \quad (2.8)$$

which imply the standard free field equation for  $C$ . The matter part of the action (2.7) is simply the first order formulation of the Maxwell-type gauge theory for the  $2k$ -form  $C$ . This formulation will prove convenient for the later incorporation of a self-duality condition.

We now consider the compactification of (2.7) on a torus  $\mathbb{T}^2$ , i.e.  $M_{4k+2} = M_{4k} \times \mathbb{T}^2$ . Let  $(x, y)$  be coordinates for  $\mathbb{T}^2$  with standard identifications, i.e

$$x \sim x + 1 \quad y \sim y + 1 ; \quad (2.9)$$

the shape of the torus is then determined by its modular parameter,  $\tau$ , which will be a function on  $M_{4k}$  in the KK ansatz. This ansatz (which includes a consistent

truncation) is

$$\begin{aligned}
ds^2(M_{4k+2}) &= ds^2(M_{4k}) + \frac{1}{\text{Im}\tau} (|\tau|^2 dy^2 + 2\text{Re}\tau dx dy + dx^2) \\
H &= G dy + F dx \\
C &= B dy + A dx ,
\end{aligned} \tag{2.10}$$

where  $F, G$  are  $(2k)$ -forms on  $M_{4k}$ , while  $A, B$  are  $(2k-1)$ -forms and  $\tau$  is a complex scalar field. A feature of this KK ansatz is that the volume of the torus is unity. More generally one could take the volume to be arbitrary, which would introduce an additional scalar field on  $M_{4k}$  in the KK ansatz. Effectively we have set this field to the constant corresponding to unit volume. This is a consistent truncation provided that we keep only that part of the gauge field action which is conformally invariant in  $4k$  dimensions, as we have done, because the volume field of the torus does not couple to the conformally invariant part of the action.

The  $4k$ -dimensional action resulting from the above KK ansatz is

$$\begin{aligned}
I_{(4k)} &= \int d^{4k}x \sqrt{-g} \left[ R - \frac{1}{2} \frac{d\tau d\bar{\tau}}{(\text{Im}\tau)^2} \right] + \alpha \int \left\{ (dA \wedge G - dB \wedge F) \right. \\
&\quad \left. + \frac{1}{2\text{Im}\tau} (G \wedge *G + 2\text{Re}\tau F \wedge *G + |\tau|^2 F \wedge *F) \right\} .
\end{aligned} \tag{2.11}$$

Following [2] we now take into account a self-duality condition on the original field-strength  $H$  by imposing the constraint

$$i_V(H - dC) = 0 , \tag{2.12}$$

where  $V = \partial/\partial x$ . From the KK ansatz we see that this gives

$$F = dA . \tag{2.13}$$

The  $2k$ -form  $G$  in (2.11) is auxiliary and may be eliminated by its field equation. Taking into account that  $F = dA$  this is just  $G = \text{Im}\tau *F - \text{Re}\tau F$ , as in (2.2).

Using this in (2.11), and dropping the  $dB \wedge F$  term which is now a total derivative, we get precisely the action (2.1). As a check, we observe that if we substitute for  $G$  in terms of  $F$  in the KK ansatz (2.10), we discover that the  $(2k+1)$ -form field strength  $H$  is self-dual.

### 3. Dyonic p-branes from self-dual (p+1)-branes

We now turn to the applications of the reduction procedure just described for  $k=1$  and  $k=2$ . For  $k=1$  the action (2.7) is a consistent truncation of the N=4 chiral D=6 supergravity theory (which is itself a truncation of the anomaly-free effective N=4 chiral supergravity obtained by compactification of IIB supergravity on  $K_3$ ). Using the KK ansatz (2.10) and setting

$$\tau = 2\rho + ie^{-2\sigma} , \quad (3.1)$$

we find the D=4 action

$$I = \int \left\{ \sqrt{-g} [R - 2\partial_\mu \sigma \partial^\mu \sigma - 2e^{4\sigma} \partial_\mu \rho \partial^\mu \rho - e^{-2\sigma} F_{\alpha\beta} F^{\alpha\beta}] - \varepsilon^{\mu\nu\alpha\beta} \rho F_{\mu\nu} F_{\alpha\beta} \right\} . \quad (3.2)$$

where  $F_{\mu\nu}$  are the components of the 2-form field strength  $F_2$ .

The field equations of this action have the following extreme charged black hole solutions [11,12]:

$$\begin{aligned} ds_{(4)}^2 &= -H^{-1} dt^2 + H ds^2(\mathbb{E}^3) \\ F_2 &= \frac{1}{\sqrt{2}} \left[ \cos \xi (\star dH) + \sin \xi dH^{-1} \wedge dt \right] \\ \tau &= \frac{\sin(2\xi)(1-H^2) + 2iH}{2(\sin^2 \xi + H^2 \cos^2 \xi)} , \end{aligned} \quad (3.3)$$

where  $\xi$  is an angle,  $H$  is a harmonic function on  $\mathbb{E}^3$  and  $\star$  is the Hodge star of  $\mathbb{E}^3$ . We have chosen the asymptotic value  $\langle \tau \rangle$  of the complex scalar field  $\tau$  to be  $i$ .



The angle  $\xi$  is the parameter of a  $U(1)$  rotation of the purely magnetic solution. The existence of this one-parameter family of ‘classical dyons’ follows from the  $Sl(2; \mathbb{R})$  invariance of the action and the existence of a  $U(1)$  subgroup of  $Sl(2; \mathbb{R})$  that leaves invariant the asymptotic value  $\langle \tau \rangle$ , the  $U(1)$  subgroup depending on  $\langle \tau \rangle$ . In the quantum theory this  $U(1)$  group is broken to  $\mathbb{Z}_2$  by the Dirac quantization condition. In particular, when  $\langle \tau \rangle = i$  the only surviving values of  $\xi$  are those for which  $\cos \xi = 0, 1$ . It might appear from this that the classical dyon solution (3.3) is irrelevant in the quantum theory. This would indeed be true if the scalar field target space  $\mathcal{M}$  were the homogeneous space  $Sl(2; \mathbb{R})/U(1)$ , as is the case *locally*, but in the string theory context S-duality implies that

$$\mathcal{M} = Sl(2; \mathbb{Z}) \backslash Sl(2; \mathbb{R})/U(1) . \quad (3.4)$$

This means that the asymptotic value  $\langle \tau \rangle = i$  should not be distinguished from any of the other values obtained from it by an  $Sl(2; \mathbb{Z})$  transformation, but it follows from (2.5) that such a transformation takes a purely magnetic solution, with (magnetic, electric) charge vector  $(1, 0)$  to a dyon with (magnetic, electric) charge vector  $(d, -b)$  where  $a, b, c, d$  are *integer* entries of the matrix  $A$  of (2.6) satisfying  $ad - bc = 1$  (so that now  $A \in Sl(2; \mathbb{Z})$ ).

To obtain the D=6 interpretation of these dyons we shall need the explicit form of the  $Sl(2; \mathbb{Z})$  transformed solution. Starting from the  $\xi = 0$  case of (3.3) an  $Sl(2; \mathbb{Z})$  transformation leads to

$$\begin{aligned} ds_{(4)}^2 &= -H^{-1} dt^2 + H ds^2(\mathbb{E}^3) \\ F_2 &= \frac{1}{\sqrt{2}} e^{\langle \sigma \rangle} [\cos \psi \star dH + \sin \psi dH^{-1} \wedge dt] \\ \tau &= 2 \langle \rho \rangle + e^{-2\langle \sigma \rangle} \left[ \frac{\sin(2\psi)(1 - H^2) + 2iH}{2(\sin^2 \psi + H^2 \cos^2 \psi)} \right] , \end{aligned} \quad (3.5)$$

where  $H$  is a harmonic function on  $\mathbb{E}^3$ . The asymptotic value of  $\tau$  is now

$$\langle \tau \rangle = \frac{bd + ac + i}{c^2 + d^2} , \quad (3.6)$$

and

$$\tan \psi = c/d , \quad (3.7)$$

where  $a, b, c, d$  are the integer entries of the  $Sl(2; \mathbb{Z})$  matrix  $A$ .

Referring to the KK ansatz (2.10), one sees that the solution lifts to the D=6 self-dual string solution

$$\begin{aligned} ds_{(6)}^2 &= H^{-1}(-dt^2 + dv^2) + H[ds^2(\mathbb{E}^3) + du^2] \\ F_3 &= \frac{1}{\sqrt{2}} \left[ (\star dH) \wedge du + dH^{-1} \wedge \epsilon(\mathbb{M}^2) \wedge dv \right] , \end{aligned} \quad (3.8)$$

where  $\epsilon(\mathbb{M}^2)$  is the volume form on the two-dimensional Minkowski spacetime  $\mathbb{M}^2$  with coordinates  $(t, v)$ , provided that the coordinates  $(u, v)$  are related to  $(x, y)$  of the KK ansatz by

$$(y, x) = (v, u)A^{-1} . \quad (3.9)$$

The configuration (3.8) is precisely the self-dual string solution of D=6 N=4 supergravity with a choice of harmonic function consistent with the KK ansatz. Clearly,  $v$  must be identified with the string's spatial coordinate. We can choose to identify it with unit period, so that  $\oint dv = 1$  where the integral is over one period. The winding numbers of the string around the  $x$  and  $y$  directions of the torus  $\mathbb{T}^2$  are then

$$\begin{aligned} (\oint dy, \oint dx) &= (\oint dv \frac{\partial y}{\partial v}, \oint dv \frac{\partial x}{\partial v}) \\ &= (d, -b) , \end{aligned} \quad (3.10)$$

which is also the (magnetic, electric) charge vector of the D=4 dyon.

The solution (3.3) is a special case of a more general extreme dyonic black hole depending on two harmonic functions [11], but this more general solution preserves only 1/4 of the N=4 supersymmetry and lifts to a self-dual string solution preserving only 1/4 of the supersymmetry of N=4 chiral D=6 supergravity. Such a D=6 string cannot be interpreted as a IIB 3-brane wrapped around a homology cycle of

$K_3$  because the latter configuration preserves 1/2 the supersymmetry. However, a D=6 self-dual string preserving 1/4 of the N=4 supersymmetry has a D=10 interpretation as two IIB 3-branes intersecting on a string [13], with the four ‘relative transverse dimensions’ (in the terminology of [14]) compactified on two 2-cycles of  $K_3$ . We shall not have anything further to say here about dyons preserving 1/4 of the supersymmetry except to point out that their existence is directly related to the existence of an N=2 supergravity, in which context they preserve 1/2 the supersymmetry. This is to be contrasted with the situation pertaining to D=8 dyonic membranes, to be discussed below. Since the minimal D=8 supergravity theory admitting supersymmetric dyonic membranes is actually maximally supersymmetric, the general supersymmetric dyonic membrane can depend on only one harmonic function. We now turn to the D=10 IIB interpretation of these solutions.

For  $k = 2$  the action (2.7) is, if the equations of motion are augmented by the self-duality constraint, a consistent truncation of the action of D=10 IIB supergravity. Applying the reduction procedure and setting  $\tau = 2\rho + ie^{-2\sigma}$ , as before, we obtain the following D=8 action:

$$I = \int \left\{ \sqrt{-g} \left[ R - 2\partial_\mu \sigma \partial^\mu \sigma - 2e^{4\sigma} \partial_\mu \rho \partial^\mu \rho - \frac{1}{12} e^{-2\sigma} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} \right] - \frac{1}{144} \epsilon^{\mu\nu\rho\sigma\alpha\beta\gamma\delta} \rho F_{\mu\nu\rho\sigma} F_{\alpha\beta\gamma\delta} \right\}. \quad (3.11)$$

This is just the action used in [5], where the field equations were shown to admit the following ‘classical’ dyonic membrane solutions:

$$\begin{aligned} ds_{(8)}^2 &= H^{-\frac{1}{2}} ds^2(\mathbb{M}^3) + H^{\frac{1}{2}} ds^2(\mathbb{E}^5) \\ F_4 &= \frac{1}{2} \cos \xi (\star dH) + \frac{1}{2} \sin \xi dH^{-1} \wedge \epsilon(\mathbb{M}^3) \\ \tau &= \frac{\sin(2\xi)(1-H) + 2iH^{1/2}}{2(\sin^2 \xi + H \cos^2 \xi)}, \end{aligned} \quad (3.12)$$

where  $ds^2(\mathbb{M}^3)$  is the 3-dimensional Minkowski metric,  $\epsilon(\mathbb{M}^3)$  is the volume form

of  $\mathbb{M}^3$ ,  $\xi$  is an angle,  $H$  is a harmonic function on  $\mathbb{E}^5$  and star is the Hodge star in  $\mathbb{E}^5$ .

To find the D=8 dyonic membranes relevant to the quantum theory we proceed as for the D=4 dyons. We first set  $\cos \xi = 1$  thereby selecting the purely magnetic solution [4], to which we assign (magnetic, electric) charge vector  $(1, 0)$ , and we then perform an  $Sl(2; \mathbb{Z})$  transformation to arrive at a solution with (magnetic, electric) charge vector  $(d, -b)$  for co-prime integers  $b, d$ . The  $Sl(2; \mathbb{Z})$ -transformed solution has the same (Einstein) metric as in (3.12), but the other fields are now

$$\begin{aligned} F_4 &= \frac{1}{2} e^{<\sigma>} \left[ \cos \psi (\star dH) + \sin \psi dH^{-1} \wedge \epsilon(\mathbb{M}^3) \right] \\ \tau &= 2 <\rho> + e^{-2<\sigma>} \cdot \frac{\sin(2\psi)(1 - H) + 2iH^{1/2}}{2(\sin^2 \psi + H \cos^2 \psi)} , \end{aligned} \quad (3.13)$$

where the vacuum values of  $\rho$  and  $\sigma$  are given by (3.6) and the angle  $\psi$  is again given by (3.7).

Using the ansatz (2.10) for  $k = 2$  we deduce that these dyonic membrane solutions lift to the following solutions in D=10:

$$\begin{aligned} ds_{(10)}^2 &= H^{-\frac{1}{2}}(ds^2(\mathbb{M}^3) + dv^2) + H^{\frac{1}{2}}(ds^2(\mathbb{E}^5) + du^2) \\ F_5 &= \frac{1}{2}(\star dH) \wedge du + \frac{1}{2}dH^{-1} \wedge \epsilon(\mathbb{M}^3) \wedge dv , \end{aligned} \quad (3.14)$$

where the coordinates  $(u, v)$  are again related to  $(x, y)$  as in (3.9). The configuration (3.14) is precisely the self-dual 3-brane solution of IIB supergravity except that  $H$  is a harmonic function on  $\mathbb{R}^5$  rather than  $\mathbb{R}^6$  because of consistency with the KK ansatz. The winding numbers of this 3-brane around the KK torus, parameterized by  $(x, y)$ , is given by the same computation as before, with the result that a dyonic membrane with (magnetic, electric) charge vector  $(m, n)$  is a 3-brane wound  $m$  times around one fundamental homology cycle of the KK torus and  $n$  times around the other one.

#### 4. Dyonic membranes as membrane-fourbrane bound states

In section 3, we showed that the D=8 dyonic membrane solutions could be lifted to D=10 as solutions of IIB supergravity. We now turn to their IIA interpretation. This question was already partly addressed in [5] since it was shown there that the dyonic membrane solutions could be lifted to D=11 as new solutions of D=11 supergravity interpolating between the membrane and the 5-brane solution. Such solutions can be viewed as special cases of orthogonally intersecting p-branes in which one brane lies entirely within the other. Thus the D=11 solution under discussion could be interpreted as a membrane within a 5-brane, i.e. (2|2, 5) in the notation of [9] where the last set of numbers indicate the values of  $p$  for the intersecting  $p$ -branes and the first number gives the value of  $p$  for the common intersection. By lifting to D=10 rather than D=11 we instead find new solutions of the D=10 IIA supergravity that interpolate between the membrane and the 4-brane solutions, i.e. (2|2, 4). Given that the D=11 result is already known, it is actually simpler to reduce from D=11 rather than lift from D=8, and we shall adopt that procedure here. Thus, our starting point will be the following D=11 (2|2, 5) solution [5]:

$$\begin{aligned}
ds_{(11)}^2 &= H^{-\frac{2}{3}}[\sin^2 \xi + H \cos^2 \xi]^{\frac{1}{3}} ds^2(\mathbb{M}^3) + H^{\frac{1}{3}}[\sin^2 \xi + H \cos^2 \xi]^{\frac{1}{3}} ds(\mathbb{E}^3) \\
&\quad + H^{\frac{1}{3}}[\sin^2 \xi + H \cos^2 \xi]^{-\frac{2}{3}} ds^2(\mathbb{E}^5) \\
F_4^{(11)} &= \frac{1}{2} \cos \xi \star dH + \frac{1}{2} \sin \xi dH^{-1} \wedge \epsilon(\mathbb{M}^3) \\
&\quad + \frac{3 \sin 2\xi}{2[\sin^2 \xi + H \cos^2 \xi]^2} dH \wedge \epsilon(\mathbb{E}^3) ,
\end{aligned} \tag{4.1}$$

where  $H$  is a harmonic function on  $\mathbb{E}^5$ , and  $\epsilon(\mathbb{M}^3)$  and  $\epsilon(\mathbb{E}^3)$  are the volume forms on  $\mathbb{M}^3$  and  $\mathbb{E}^3$ , respectively.

We use the following KK ansatz for reducing, and truncating, D=11 supergravity to D=10, with the D=10 metric in the string frame:

$$\begin{aligned}
ds_{(11)}^2 &= e^{-\frac{2}{3}\phi} ds_{(10)}^2 + e^{\frac{4}{3}\phi} du^2 \\
F_4^{(11)} &= F_4 + F_3 \wedge du .
\end{aligned} \tag{4.2}$$

Using this ansatz there are still three distinct ways of reducing the D=11 (2|2, 5) solution to D=10, depending on the choice of compactifying direction. The corresponding D=10 solutions have the interpretation as a membrane within a 4-brane (2|2, 4), a membrane within a 5-brane, (2|2, 5) and a string within a 4-brane (1|1, 4). As explained above, the case of immediate interest is the first, and using the ansatz (4.2) we find the following new (2|2, 4) solution of D=10 IIA supergravity:

$$\begin{aligned}
ds_{(10)}^2 &= H^{-\frac{1}{2}} ds^2(\mathbb{M}^3) + \frac{H^{\frac{1}{2}}}{\sin^2 \xi + H \cos^2 \xi} ds^2(\mathbb{E}^2) + H^{\frac{1}{2}} ds^2(\mathbb{E}^5) \\
F_4 &= \frac{1}{2} \cos \xi \star dH + \frac{1}{2} \sin \xi dH^{-1} \wedge \epsilon(\mathbb{M}^3) \\
F_3 &= \frac{3}{2} \frac{\sin(2\xi)}{[\sin^2 \xi + H \cos^2 \xi]^2} \epsilon(\mathbb{E}^2) \wedge dH \\
e^{4\phi} &= \frac{H}{[\sin^2 \xi + H \cos^2 \xi]^2} .
\end{aligned} \tag{4.3}$$

This configuration is presumably the field-theoretic realization of a IIA superstring D-2-brane within a D-4-brane. Remarkably, the IIA (2|2, 5) solution is self-dual in the sense that the procedure used in [14] to construct a dual solution yields the same solution.

We conclude by considering the other two possible reductions of the D=11 (2|2, 5) solution. Using the same KK ansatz but reducing in the other two ways, as explained above, we find, firstly, the following D=10 IIA (1|1, 4) solution:

$$\begin{aligned}
ds_{(10)}^2 &= H^{-1} [\sin^2 \xi + H \cos^2 \xi]^{\frac{1}{2}} ds^2(\mathbb{M}^2) + [\sin^2 \xi + H \cos^2 \xi]^{-\frac{1}{2}} ds^2(\mathbb{E}^3) \\
&\quad + [\sin^2 \xi + H \cos^2 \xi]^{\frac{1}{2}} ds^2(\mathbb{E}^5) \\
F_4 &= \frac{1}{2} \cos \xi \star dH - \frac{3}{2} \frac{\sin(2\xi)}{[\sin^2 \xi + H \cos^2 \xi]^2} \epsilon(\mathbb{E}^3) \wedge dH \\
F_3 &= \frac{1}{2} \sin \xi dH^{-1} \wedge \epsilon(\mathbb{M}^2) \\
e^{4\phi} &= \frac{[\sin^2 \xi + H \cos^2 \xi]}{H^2} .
\end{aligned} \tag{4.4}$$

This is presumably the field theoretic realization of a bound state of fundamen-

tal IIA string with a D-4-brane, perhaps analogous to the bound states of IIB fundamental strings with D-1-branes discussed in [15].

Secondly, we find the following D=10 IIA (2|2, 5) solution:

$$\begin{aligned}
ds_{(10)}^2 &= H^{-\frac{1}{2}}(\sin^2 \xi + H \cos^2 \xi)^{\frac{1}{2}} ds^2(\mathbb{M}^3) + H^{\frac{1}{2}}(\sin^2 \xi + H \cos^2 \xi)^{-\frac{1}{2}} ds^2(\mathbb{E}^3) \\
&\quad + H^{\frac{1}{2}}(\sin^2 \xi + H \cos^2 \xi)^{\frac{1}{2}} ds^2(\mathbb{E}^4) \\
F_4 &= -\frac{3}{2} \frac{\sin(2\xi)}{[\sin^2 \xi + H \cos^2 \xi]^2} \epsilon(\mathbb{E}^3) \wedge dH + \frac{1}{2} \sin \xi dH^{-1} \wedge \epsilon(\mathbb{M}^3) \\
F_3 &= \frac{1}{2} \cos \xi \star dH \\
e^{4\phi} &= H[\sin^2 \xi + H \cos^2 \xi] .
\end{aligned} \tag{4.5}$$

This is just the magnetic dual of (4.4).

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